Recitation Session 5

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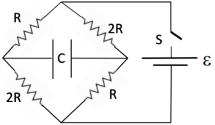
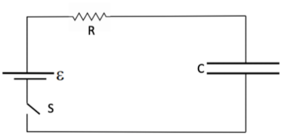
1. A resistor of resistance R outputs 50W of power when connected across a potential difference of 100V.
   (a) What is the value of the resistance R?
   (b) Now the resistor is disconnected from the 100V source and then connected to a battery of E.M.F 5V. What is the current through the resistor and the power output of the resistor in this situation?

2. Consider the circuit shown below: a battery with e.m.f $\mathcal{E}$ is connected to an arrangement of a capacitor with capacitance C and four resistors of resistances R and $2R$. A switch S connects the battery to the circuit.



(a) At time $t = 0$ the switch is connected to the circuit. The instant after the switch is connected what are the currents through each of the resistors?
(b) After we wait for a very long time, and the circuit reaches equilibrium, what are the currents through and potential differences across each of the resistors?
(c) What is the charge on the capacitor C after we have waited for a long time after the switch has been closed? Indicate which end of the capacitor has the positive charge.
(d) After the capacitor C is fully charged, the switch S is opened again. What is the time constant for the decay of the capacitor charge?
(e) What would be the final charge of the capacitor C if all the resistors were equal and had resistance R?

3. A consider a series RC circuit where a capacitor C is connected to a battery with e.m.f $\mathcal{E}$ and a resistor R as shown below.



At time $t = 0$ the switch S is closed. Your goal is to compute the energy produced/dissipated through each circuit element after the switch has been closed for a very long time ($t \to \infty$).

(a) Starting with the equation $\Delta U = \int V dq$ find the energy delivered by the battery to the circuit. Show that this energy is exactly double the final energy of the capacitor.

(b) Now explicitly compute the integral $U_R = \int_0^\infty i^2(t) R dt = \int_0^\infty \frac{V_R^2(t)}{R} dt$ to show that the energy dissipated trough the resistor is indeed exactly half that the battery produced. Note we are integrating the power through the resistor of all time to compute the total energy dissipation. You should choose which of the two equivalent integrals you wish to compute and then compute either $i(t)$ or $V_R(t)$. For practice compute both $i(t)$ and $V_R(t)$ but choose one to evaluate either the first or the second integral.

Moral: No matter the resistance and capacitance values, in a series RC circuit, exactly half the energy produced by a battery is dissipated through resistance, and exactly half of it gets stored in a capacitor.

4. A circuit consists of a resistor with resistance R and a capacitor of capacitance C connected in series. The capacitor is first fully charged and then allowed to discharge. The capacitor takes 2 seconds to reduce its charge to half its initial value during its discharge. What is the time in which the energy in the capacitor reduces to half its initial value?