Recitation Session 2

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Desiderata

Desiderata

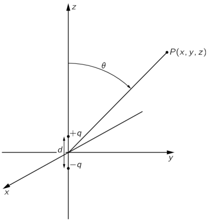
1. Electric Dipole
   - Torque
   - Energy
2. Electric Flux, Φ_E
3. Gauss' Law
   - Cylindrical symmetry
   - Planar symmetry
   - Spherical symmetry
4. Food for thought

Electric Dipole- Definition

- Two equal and opposite charge separated a by a small distance forms an electric dipole. Eg: Polar molecules like HCl, KCl, etc
- Electric dipole moment is defined by:

$$\vec{p} = q\vec{d}$$

Here magnitude of $\vec{d}$ is the distance between the charges and the direction of $\vec{d}$ is from negative charge to the positive charge



A dipole: two charges $+q$ and $-q$ the distance d apart.

Torque

- An electric dipole placed in uniform electric field experiences a torque given by:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Energy

- An electric dipole placed in uniform electric field has a potential energy given by:

$$U = -\vec{p} \cdot \vec{E}$$

- The energy is minimum($U = -pE$) when the dipole points in the direction of field and maximum when it points opposite to the direction of field($U = +pE$)

Discussion Problem 1

1. Consider a dipole with dipole moment $\vec{p} = p_0(3\hat{i} + 4\hat{j})$ where p_0 is a constant. A constant electric field $\vec{E} = E_0\hat{i}$ exists in the region where the dipole is placed.

   (a) What is the magnitude of the dipole moment of the dipole?

   (b) What is the net electric force on the dipole?

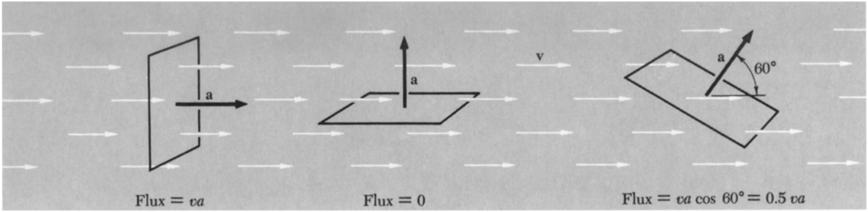
   (c) What is the net electric torque on the dipole?

   (d) If the dipole is rotated such that the final dipole moment vector is given by, $\vec{p} = p_0(-4\hat{i} + 3\hat{j})$, what was the work done on the dipole? (Hint: What is the change of potential energy of the dipole?)

Desiderata

Flux-Introduction

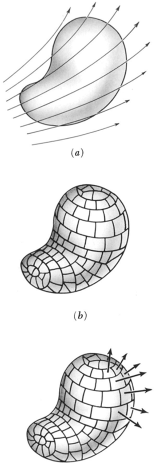
- Electric flux is the rate of flow of the electric field through a given area



Flux- area as a vector

- Area is vector! Magnitude is area of the surface. Direction is arbitrarily chosen for open surface and chosen to be outwards for closed surfaces.
- For any arbitrary shape, electric flux is defined by

$$\phi_E = \iint \vec{E} \cdot d\vec{A}$$



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Gauss' Law: Background

- Gauss' Law states:

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

- It allows us to *easily* calculate electric field for problems with: spherical, cylindrical or planar symmetry.

Applying Gauss' Law

Steps involved in solving problems using Gauss' Law:

- Use symmetry to determine direction of electric field
- Draw Gaussian surface: cylindrical,planar or spherical
- Find charge inside the Gaussian surface using:
  - Line charge: $q_{enc} = \int \lambda dl$, λ is charge/length
  - Surface charge: $q_{enc} = \int \sigma da$, σ is charge/area
  - Volume charge: $q_{enc} = \int \rho dV$, ρ is charge/volume
- Evaluate integral on Gaussian surface. Remember:
  - Where $\vec{E} \perp d\vec{A}$, $\vec{E} \cdot d\vec{A} = 0$
  - Where $\vec{E} \parallel d\vec{A}, \vec{E} \cdot d\vec{A} = |\vec{E}||d\vec{A}|$

Electric field due to infinite line charge

What is the electric field at a distance r from a positively charged line charge, with charge density λ?

Electric field due to sheet of charge

What is the electric field at a distance r from a positively charged, uniform plane sheet of charge, with charge per unit area σ?

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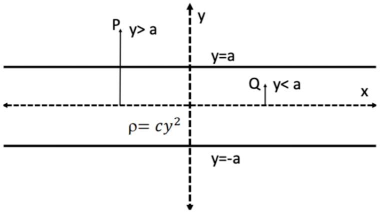
Discussion Problem 2

2. Consider a slab whose faces are infinite planes in the xz plane and the thickness of the slab is $2a$ along the y axis as shown below in the diagram. The slab is a variable volume charge density $\rho = cy^2$ where c is a positive constant.



i. Argue that by symmetry the electric field will not depend on x and z coordinates, will point vertically up or down along the y axis, and should be the same at any y and $-y$.

ii. Use the symmetry arguments above in conjunction with Gauss Law to compute the magnitude of the electric field due to the slab at the points P which is at any x, z with $|y| \geq a$ and at the point Q which is inside the slab, again at any xz but with $|y| \leq a$.

Electric field due to shell of charge Q

What is the electric field at a distance r from a positively charged shell with charge Q?

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Electric field due to uniformly charged sphere

What is the electric field at a distance r from a positively charged sphere, with charge per unit volume $\rho = \rho_0$?

Applying Gauss' Law

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Discussion Problem 3

3. According to quantum mechanics, a particle, unless we measure its position, has no definite position and has a finite probability of being anywhere in space. For the hydrogen atom– which has one proton and one electron– we model the atom to be a sphere with the electron "smeared" in a spherically symmetric cloud around a pointlike nucleus at the centre with positive charge e. For the lowest energy state of the electron, which is called the "ground state", the charge distribution of the electron is given by,

   $\rho(r) = \frac{-8e}{\pi}e^{-4r}$

   where e is the magnitude of the electronic charge the distance r is measured in a set of units called Angstrom (A) (10^{-10}m)which is popular in atomic physics. The "average" radius of the hydrogen atom is called the Bohr radius which is equal to roughly $0.5A$.

   Your goal is to compute the net electric field inside the atom.

   (a) Consider an appropriate Gaussian surface at a distance r from the nucleus. Find the total charge enclosed in it by integrating the charge density, i.e evaluating the integral $q_{in} = \int \rho dV$. Recall that $dV = 4\pi r^2 dr$ in spherical coordinates.

   (b) For what value of r is the total charge due to electron cloud(excluding the proton)equal to $-e$? Does the answer make sense?

   (c) Now compute the net electric field at distance r from the nucleus. Do not forget the proton!

   You will find the following integral useful,

   $\int u^2 e^{-u} du = -(u^2 + 2u + 2)e^{-u}$

Desiderata

Experimental verification of Coulomb's law

- People have assumed a form for Coulomb's law like:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^{2+\delta}}$$

On laboratory scales, it has been shown $\delta < 10^{-15}$

- No deviation has been found even at the scale of fraction of the size of the atomic nucleus 10^{-16}m
- Space probes have validated Coulomb's law at scale of 10^7km
- How to test Coulomb's law at astronomical length scale remains an open problem
- The coefficient $\frac{1}{4\pi\epsilon_0}$ is constant until the nuclear scale(10^{-15}m) when it starts to increase

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Photographing the atom

- First ever picture of an atom was taken in 1955 using field-emission microscope which works on principles of electrostatics. Create a high electric field, tungsten electrons (or He ions) are attracted to periphery and detected. The resolution is high enough to resolve position of individual atoms on tip of the needle