

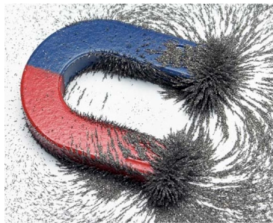
# Recitation Session 1

Achint Kumar

Duke University

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# Welcome to PHYS 152



# My Background

- My name is Achint Kumar. I am a 5th year PhD candidate in Physics Department.
- I have an undergraduate degree in physics and electrical engineering
- My research is in machine learning and computational neuroscience. My advisor is Prof. John Pearson (ECE) and Prof. Richard Mooney (Neurobiology)
- In my research, I am trying to understand how the brain forms a representation of nature by integrating information from various senses

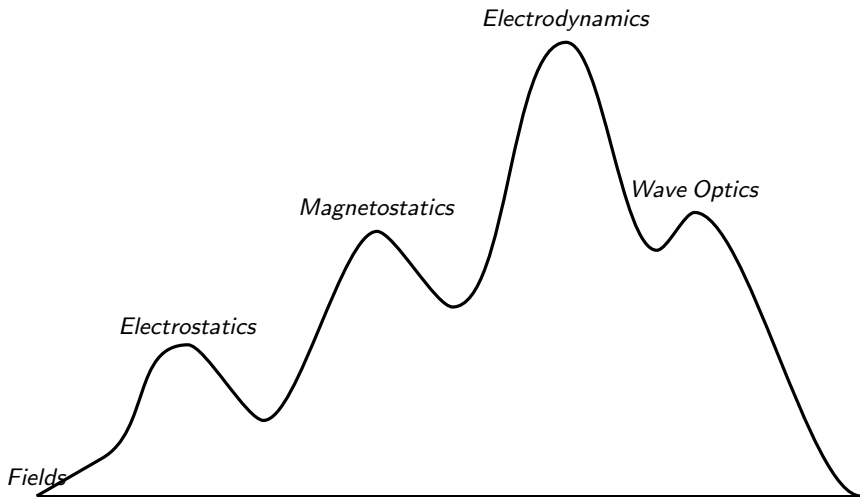
# Structure of the recitation

- 20-25 minutes: Overview of lecture, introducing new concepts
- 1 hour: Solving discussion problems
- 20 minutes: Quiz. With prior permission you can get extra time.

Please feel free to contact me at: [achint.kumar@duke.edu](mailto:achint.kumar@duke.edu).

The slides will be available on my website:  
[achintzeus1994@github.io](mailto:achintzeus1994@github.io)

# Climbing Mount Electromagnetism



# Motivation

*I KEEP six honest serving-men  
(They taught me all I knew);  
Their names are What and Why and When  
And How and Where and Who.*

—Rudyard Kipling

# What?

## What would it mean to climb Mt. Electromagnetism?

- Understand the physical content of Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

# Why?

## Why climb Mt. Electromagnetism?

- ALL of electromagnetic phenomena results from Maxwell's equations.
- Idea of field is pervasive throughout physics and engineering. You'll encounter this idea again in fluid mechanics, thermodynamics, etc
- The view from the top is stunning. You will appreciate the unification of three disparate phenomena: electric, magnetic, optical. The nature of light will be revealed to electromagnetic.



# How?

## How to climb Mt. Electromagnetism?

- Electromagnetism is a highly visual subject. Think in pictures
- Be curious. Try to connect what you learn to your everyday experience. Ask questions!
- Solve variety of problems!

# Coulomb's law

- Magnitude of force between two charges  $q_1$  and  $q_2$  separated by a distance  $r$  (in vacuum) is given by,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

- Like charges repel, unlike charges attract
- Electrostatic force is  $10^{36}$  times stronger than gravity
- This "action at a distance" approach works only for static charges. A more abstract framework based on *fields* works even for moving charges.

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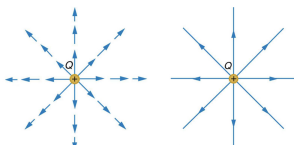
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# Field

- Instead of direct interaction, we say a charge creates a field around it which is experienced by other charges in its vicinity. This field leads to the force. Mathematically, a field is a function of space and time.

$$\vec{F} = q\vec{E}(\vec{x}, t)$$

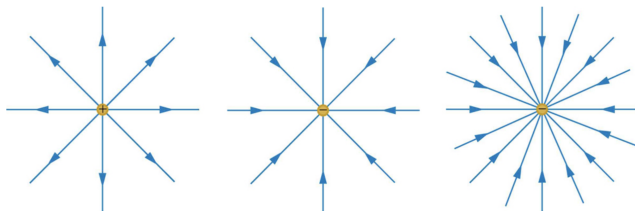
- Static electric charge creates *electric field* (a vector field) around it . It is the force felt by unit charge when it is brought near it.



Equivalent representation of electric field

# Static electric fields are radial

- Electric field lines go *radially* outward for positive charge and go radially inward for negative charge
- Higher density of field lines imply greater strength



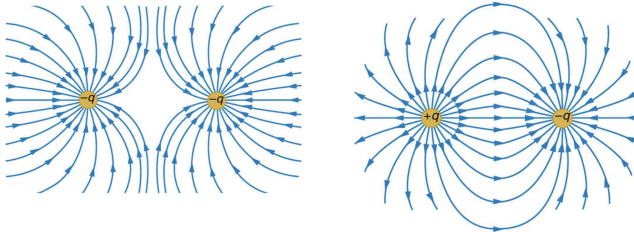
Field lines for positive charge and negative charges of different magnitude

# Electric fields obeys superposition principle

- Electric field obeys *superposition principle*. This means, if electric field of a collection of charges is the sum of electric field due to isolated charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

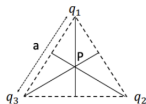
- So, does magnetic field. Not all fields obey superposition. Eg: velocity field of fluids



Field lines for positive charge and negative charges of different magnitude

# Problem 1

1. Consider three charges  $q_1$ ,  $q_2$  and  $q_3$  arranged at the vertices of an equilateral triangle of side  $a$  as shown below. The point  $P$  is the centroid of the triangle.

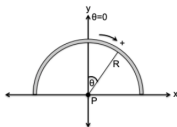


- i. By explicit computation show that if all the charges are equal then the net electric field at the point  $P$  is zero.
- ii. If you place a point charge  $Q$  at the point  $P$  it will be at a point of equilibrium. Is this equilibrium stable or unstable? Discuss how the nature of the equilibrium depends on the sign of the charge  $Q$ . Does the nature of equilibrium depend on in which direction one moves the charge  $Q$ ?



## Problem 2

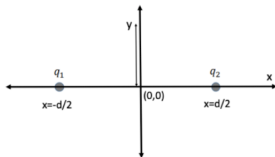
2. A thin nonconducting rod is bent in the shape of a semi-circle of radius  $R$  as shown. The rod has a linear charge density given by  $\lambda = \lambda_0 \sin \theta$  where  $\theta$  is the angle measured from the  $y$  axis as shown, and  $\lambda_0$  is a positive constant.



Find the direction and the magnitude of the electric field at point  $P$  which is at the origin.

## Problem 3a

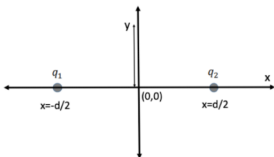
3. Two charges  $q_1$  and  $q_2$  are placed on the  $x$  axis at  $x = \frac{d}{2}$  and  $x = -\frac{d}{2}$  as shown below. The following questions pertain to electric field due to different combinations of the two charges.



- (a) Consider the case where  $q_1 = 3q$  and  $q_2 = q$  where  $q$  is a positive constant. Find the location of all point/ points on the  $x$  axis where the net electric field due to both charges is zero.

## Problem 3b

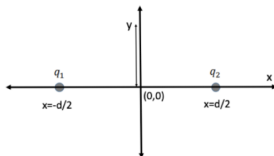
3. Two charges  $q_1$  and  $q_2$  are placed on the  $x$  axis at  $x = \frac{d}{2}$  and  $x = -\frac{d}{2}$  as shown below. The following questions pertain to electric field due to different combinations of the two charges.



- (b) Now consider the case where  $q_1 = 3q$  and  $q_2 = -q$ . Find the location of all point/points on the  $x$  axis where the net electric field due to both charges is zero.

## Problem 3c

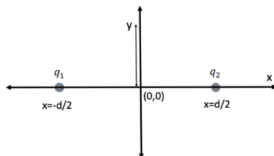
3. Two charges  $q_1$  and  $q_2$  are placed on the  $x$  axis at  $x = \frac{d}{2}$  and  $x = -\frac{d}{2}$  as shown below. The following questions pertain to electric field due to different combinations of the two charges.



- (c) Consider the case where  $q_1 = q_2 = q$ . Find the direction and magnitude of the net electric field due to this charge distribution at an arbitrary  $y > 0$  on the  $y$  axis. Indicate the direction of the field by a unit vector. ( $\vec{E} = \frac{qy}{2\pi\epsilon_0(y^2 + \frac{d^2}{4})^{\frac{3}{2}}} \hat{j}$ )

## Problem 3d

3. Two charges  $q_1$  and  $q_2$  are placed on the  $x$  axis at  $x = \frac{d}{2}$  and  $x = -\frac{d}{2}$  as shown below. The following questions pertain to electric field due to different combinations of the two charges.



- (d) This is a continuation of part (c). Consider the case when  $y \gg d$  i.e the point on the  $y$  axis is very far away relative to the separation between the charges. Take the expression of the net field you computed in part (c) and show that the net field is approximately  $\vec{E} = \frac{2q}{4\pi\epsilon_0 y^2} \hat{j}$

## Problem 3e-f

- (e) **Field due to a dipole** A dipole is an arrangement of two equal and opposite charges  $q$  and  $-q$  separated by a distance  $d$ . You will learn about the physical situations where dipoles appear and their properties in the next lecture. A quantity called the dipole moment defined to be  $p = qd$  appears in all calculations with dipoles. For the purpose of this problem use that as a definition. We will see what is the physical significance of this quantity when we discuss dipoles. In this problem you will work out the electric field due a dipole at some locations.

Consider the case where  $q_1 = q$  and  $q_2 = -q$ , i.e a dipole with magnitude of the dipole moment  $p = qd$ . Compute the magnitude and direction of the electric field due to the dipole at an arbitrary  $y > 0$  on the  $y$  axis. Indicate the direction of the field by a unit vector. ( $\vec{E} = \frac{p}{4\pi\epsilon_0(y^2 + \frac{d^2}{4})^{\frac{3}{2}}} \hat{i}$ )

- (f) This is a continuation of part (e). Consider the case when  $y \gg d$  i.e the point on the  $y$  axis is very far away relative to the separation between the charges. Take the expression of the net field you computed in part (e) and show that the net field is approximately  $\vec{E} = \frac{p}{4\pi\epsilon_0 y^3} \hat{i}$

# Food for thought

- Why is Coulomb's law and Newton's gravitational law both inverse squared in distance?
- Why are there negative charges but no negative mass?
- Why does the proton charge have exactly the same magnitude as electron's charge?
- Why is electric charge quantized but mass is not quantized?

# Bumble bee pollination

- Bumble bees use electric field to decide the flower to visit for pollination

## Electric fields of flowers stimulate the sensory hairs of bumble bees

Harold H. Zakon

+ See all authors and affiliations

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**Fig. 1.** A bumble bee can detect the electric fields of flowers via the deflections of many tiny mechanosensory filiform hairs on its head and body. (A) Bees accumulate positive charge on their bodies as they fly. Flowers have negative charges. The interaction of these charges when a bumble bee alights on a flower mechanically moves the bee's antennae and filiform hairs. (B) Stimulation of antennae or filiform sensory hairs with electric charge moves them. The electro-mechanical movements of the bumble bee antenna (red arrows) do not activate antennal sensory neurons, whereas movements of the filiform hairs (blue arrows) do.